OPTIMAL LYAPUNOV INEQUALITIES FOR BOUNDARY VALUES PROBLEMS

ANTONIO CAÑADA[1] AND SALVADOR VILLEGAS[2]

[1,2]Departamento de Análisis Matemático, Universidad de Granada, Spain

Abstract. The classical Lyapunov inequality states that if $a \in L^1(0,L)$, then a necessary condition for the boundary value problem

$$u''(x) + a(x)u(x) = 0\,, \quad x \in (0,L)\,, \quad u(0) = u(L) = 0$$

to have nontrivial solutions is that $\int_0^L a^+(x)\,dx > 4/L$, where $a^+(x) = \max\{a(x), 0\}$.

With this motivation, we extended this result to linear problems with Neumann and mixed boundary conditions (also to partial differential equations), establishing optimal results in terms of the L^p-norm of a^+, for every $1 \le p \le \infty$ ([1, 2]).

On the other hand, the relation between Neumann boundary conditions and disfocality arises in a natural way: under the natural conditions for Neumann problems, i.e., $a \in L^1(0,L) \setminus \{0\}, \int_0^L a \ge 0$, if u is any nontrivial solution of the problem $u'' + a\,u = 0$ in $(0,L)$, then u must have a zero $c \in (0,L)$ and, consequently, u is a solution of a mixed boundary value problem in the intervals $(0,c)$ and (c,L). This simple observation can be used to obtain optimal necessary conditions on the coefficient a for which the corresponding linear problem admits a nontrivial solution ([3]). Moreover, combining these linear results with Schauder fixed point theorem, we obtain some new results about the existence and uniqueness of solutions for resonant nonlinear problems. Finally, we show some extensions of previous results to higher eigenvalues ([4]).

References

[1] A. Cañada, J. A. Montero and S. Villegas, *Liapunov-type inequalities and Neumann boundary value problems at resonance,* Math. Inequal. Appl. 8 (2005), 459-475.

[2] A. Cañada, J. A. Montero and S. Villegas, *Lyapunov inequalities for partial differential equations,* J. Funct. Anal. 237 (2006), 176-193.

[3] A. Cañada and S. Villegas, *Optimal Lyapunov inequalities and Neumann boundary conditions using L^p norms,* Discrete Contin. Dyn. Syst. 20 (2008), 877-888.

[4] A. Cañada and S. Villegas, *Lyapunov inequalities for Neumann boundary conditions at higher eigenvalues,* submitted for publication.