

# WEIGHTED INEQUALITIES FOR INTEGRAL OPERATORS WITH VARIABLE DOMAIN OF INTEGRATION

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ABSTRACT. Let  $0 < p < \infty$ ,  $\|f\|_p := \left(\int_0^\infty |f(x)|^p dx\right)^{1/p}$  and let  $v(x) \geq 0$  be a weight. Denote  $L_{p,v}$  the weighted Lebesgue space with (quasi) norm  $\|f\|_{p,v} := \|fv\|_p$ . We study the integral operators of the form

$$Kf(x) = \int_{a(x)}^{b(x)} k(x,y)f(y)dy,$$

acting from  $L_{p,v}$  to  $L_{q,w}$ . The main attention is devoted to the case, when  $k(x,y) \equiv 1$  for which we denote

$$Hf(x) = \int_{a(x)}^{b(x)} f(y)dy$$

and the Geometric Mean Operator

$$Gf(x) := \exp\left(\frac{1}{b(x) - a(x)} \int_{a(x)}^{b(x)} \log f(y)dy\right), \quad f(y) \geq 0.$$

The border functions  $a(x)$  and  $b(x)$  satisfy the following conditions:

- (i)  $a(x)$  and  $b(x)$  are continuous and increasing on  $[0, \infty)$ ,
- (ii)  $a(x) < b(x)$  for all  $x \in (0, \infty)$ ,  $a(0) = b(0) = 0$ ,  $a(\infty) = b(\infty) = \infty$ .

The main topic is  $L_{p,v} - L_{q,w}$ - boundedness and compactness of operators  $K$ ,  $H$  and  $G$ . Some application for the two-sided estimates of the approximation numbers of operator  $H$  is given.