

BOOTSTRAPPING WEIGHTED FOURIER INEQUALITIES

GORD SINNAMON

UNIVERSITY OF WESTERN ONTARIO, LONDON, ONTARIO, CANADA

ABSTRACT. A weighted Fourier inequality expresses the boundedness of the Fourier transform considered as a map between weighted spaces. In this case, between weighted Lebesgue and Hardy spaces. Composing the Fourier transform with multiplication operators has the effect of changing the weights on these spaces. In this way, new weighted Fourier inequalities are produced from known ones.

The main difficulty encountered is in handling the convolution operators that arise when the multiplication operators are conjugated by the Fourier transform. One approach to this problem is to majorize these operators by convolution operators with positive kernels and establish the boundedness of the positive operators using variants of Schur's lemma.

A single application of this procedure has already produced new inequalities, including some with power weights. However, since the "input" is a known inequality and the "output" is a new one, the process can be iterated, using the output of the one iteration as the input to the next. Careful control of constants in these inequalities is needed to establish convergence of this iteration.