COMPARISON OF GENERALIZED QUASI-ARITHMETIC MEANS

ZSOLT PÁLES

University of Debrecen, Hungary

Abstract. The notion of quasi-arithmetic mean is one of the central concepts of the theory of means and inequalities. Various classical inequalities, such as the arithmetic-geometric mean inequality, the Cauchy–Schwarz inequality, Hölder's and Minkowski's inequalities can be treated as particular inequalities for quasi-arithmetic means.

Given an interval $I \subset \mathbb{R}$ and a strictly monotonic continuous function $f : I \to \mathbb{R}$, the $\mathcal{A}_f$-*quasi-arithmetic mean* of k elements $x_1, \dots, x_k$ of I is defined as

$$\mathcal{A}_f(x_1, \dots, x_k) := f^{-1}\left(\frac{f(x_1) + \cdots + f(x_k)}{k}\right).$$

Since $\mathcal{A}_f = \mathcal{A}_{-f}$ holds for all strictly monotonic continuous functions, hence we can always assume that f is strictly increasing. In the particular cases when $f(x)$ is of the form x, $\ln(x)$, and $1/x$, the resulting mean is the *arithmetic mean*, *geometric mean*, and *harmonic mean*, respectively.

The aim of the talk is to consider the following generalization of quasi-arithmetic means: Given an interval $I \subset \mathbb{R}$ and strictly monotone increasing continuous functions $f_1, \dots, f_k : I \to \mathbb{R}$, the *generalized quasi-arithmetic mean* $\mathcal{M}_{f_1,\dots,f_k}$ mean of k elements $x_1, \dots, x_k$ of I is defined by

$$\mathcal{M}_{f_1,\dots,f_k}(x_1, \dots, x_k) := (f_1 + \cdots + f_k)^{-1}\big(f_1(x_1) + \cdots + f_k(x_k)\big).$$

Our main results deal with the comparison problem for generalized quasi-arithmetic means. Applying a non-obvious iteration process, we obtain various necessary and sufficient conditions for the comparison of two generalized quasi-arithmetic means. Consequences, such as the solutions of the equality problem as well as the homogeneity problem of generalized quasi-arithmetic means will also be discussed.