KUHN-TYPE RESULTS FOR QUASICONVEX FUNCTIONS

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Abstract. It is known by the theorem of Kuhn that if a function is t–convex with a fixed $t \in (0,1)$, then it is $(1/2)$–convex and hence Q–convex (i.e. t–convex for every rational number $t \in [0,1]$). We consider analogous problem for quasi-convex functions.

Let D be a convex subset of a vector space and $t \in (0,1)$ be a fixed number. A function $f : D \to \mathbf{R}$ is called *t–quasiconvex* if

$$f\big(tx + (1-t)y\big) \le \max\{f(x), f(y)\}, \qquad x, y \in D. \tag{0.1}$$

We say that f is *strictly t–quasiconvex* if it satisfies (0.1) and moreover

$$f\big(tx + (1-t)y\big) < \max\{f(x), f(y)\} \qquad \text{if} \quad f(x) \ne f(y).$$

Simple examples show that t–quasiconvex functions need not be $(1/2)$– quasiconvex, and $(1/2)$–quasiconvex functions need not be Q–quasiconvex. However, every strictly t–quasiconvex function is strictly $(1/2)$–quasiconvex and, consequently, it is strictly Q–quasiconvex.