# MAXIMAL OVERDETERMINED WEIGHTED HARDY'S INEQUALITY ON A FINITE INTERVAL

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Abstract. Let $I = (a, b)$ be an interval, $k \geq 1$, $1 < p, q < \infty$. Let $u$, $v$ be weight functions such that $|u|^q$, $|v|^p$, $|v|^{-p'}$ are locally integrable on $(a, b)$. The weighted Lebesgue space is given by

$$L_{p,v} = L_{p,v,(a,b)} = \{f : \|fv\|_p = \left(\int_a^b |fv|^p\right)^{1/p} < \infty\}.$$

We consider weighted Hardy-type inequality of the form

$$\|Fu\|_q \leq C \left\|F^{(k)}v\right\|_p \tag{1}$$

on the interval $I = (a, b)$ for all functions $F(x)$ with absolutely continuous $(k-1)$th derivative $F^{(k-1)}(x)$ and such that $\left\|F^{(k)}v\right\|_p < \infty$,

$$F(a) = F'(a) = \cdots = F^{(k-1)}(a) = F(b) = \cdots = F^{(k-1)}(b) = 0. \tag{2}$$

This type of the inequality is called a maximal overdetermined Hardy's inequality of $k$th order. Investigations of the inequality (1), where a function satisfies some set of zero boundary conditions, was initiated by A. Kufner. A number of overdetermined problems were solved by P. Gurka, A. Kufner, G. Sinnamon, V.D. Stepanov (see also [1], Chapter 4). Main part of [2] is devoted to this problem in case $I = (0, \infty)$. This case is different to some extent from the case of a finite interval and the difference is such that the only one zero condition at infinity for the least derivative is important. That work contains a complete solution for $k = 2$ and also for some cases with $k > 2$ including the maximal overdetermined case on the semiaxis.

Now, we prove necessary and sufficient conditions on the weight functions $u$ and $v$ for the inequality (1), (2) to hold on interval $I = (0, 1)$ for $k > 2$. Some examples and application are also given.

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## References

[1] A. Kufner and L.-E. Persson, *Weighted inequalities of Hardy type*, World Scientific, Singapore, 2003.

[2] M. Nasyrova, *Weighted inequalities involving Hardy-type and geometric mean operators*, Doctoral Thesis 2002:03, Department of Mathematics, Luleå University of Technology, 2002.