ON EULER-TYPE IDENTITIES AND APPLICATIONS

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Abstract. This talk deals with Euler type identities proved in [1] and holding for every $x \in [a,b]$ and any $f : [a,b] \to \mathbb{R}$ such that $f^{(n-1)}$ is continuous function of bounded variation on $[a,b]$ for some $n \geq 1$:

$$f(x) = \frac{1}{b-a}\int_a^b f(t)\mathrm{d}t + T_n(x) + R_n^1(x) \tag{1}$$

$$f(x) = \frac{1}{b-a}\int_a^b f(t)\mathrm{d}t + T_{n-1}(x) + R_n^2(x), \tag{2}$$

where

$$T_m(x) = \sum_{k=1}^{m} \frac{(b-a)^{k-1}}{k!} B_k\left(\frac{x-a}{b-a}\right)\left[f^{(k-1)}(b) - f^{(k-1)}(a)\right],$$

$$R_n^1(x) = -\frac{(b-a)^{n-1}}{n!}\int_{[a,b]} B_n^*\left(\frac{x-t}{b-a}\right)\mathrm{d}f^{(n-1)}(t),$$

$$R_n^2(x) = -\frac{(b-a)^{n-1}}{n!}\int_{[a,b]} \left[B_n^*\left(\frac{x-t}{b-a}\right) - B_n\left(\frac{x-a}{b-a}\right)\right]\mathrm{d}f^{(n-1)}(t).$$

Here, $B_k(x)$, $k \geq 0$, are the Bernoulli polynomials and $B_k^*(x)$, $k \geq 0$, are periodic functions of period 1 such that $B_k^*(x) = B_k(x)$, $0 \leq x < 1$.

In the past eight years the identites (1) and (2) were investigated by Matić, Pečarić and other authors in many directions. In this talk some of those investigations are pointed out. For example, various identities generalizing clasical quadrature rules such as midpoint, trapezoid, Simpson rule etc. were obtained from (1) and (2) and estimations of accuracy for such rules were given by Dedić, Matić, Pearce and Pečarić. Also, generalizations of Hadamard's inequalities based on the identities obtained from (1) and (2) were obtained by Matić, Pečarić and Vukelić, while some Grüss type inequalities based on the identities obtained from (1) and (2) were considered by the same authors.

Furthermore, the identities (1) and (2) were generalized in the following ways:

The sequence $(B_k(x),\ k \geq 0)$ of Bernoulli polynomials can be replaced by an arbitrary sequence $(P_k(x),\ k \geq 0)$ of harmonic polynomials. This was done in [2].

The sequence $(B_k(x),\ k \geq 0)$ can be replaced by an arbitrary sequence $(P_k(x),\ k \geq 0)$ of $w-$harmonic functions with respect to some weight function $w : [a,b] \to \mathbb{R}$. This was done in [3].

More generally, the sequence $(B_k(x),\ k \geq 0)$ can be replaced by an arbitrary sequence $(P_k(x),\ k \geq 0)$ of $\mu-$harmonic functions with respect to a real Borel measure μ on $[a,b]$. This was done in [4].

References

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