

# REAL AND COMPLEX OPERATOR NORMS BETWEEN QUASI-BANACH $L^p - L^q$ -SPACES

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ABSTRACT. Any bounded linear operator between real Lebesgue spaces  $T : L^p(\mu) \rightarrow L^q(\nu)$  for  $0 < p, q \leq \infty$  has natural complex extension  $T_{\mathbf{C}} : L^p_{\mathbf{C}}(\mu) \rightarrow L^q_{\mathbf{C}}(\nu)$  given by the formula

$$T_{\mathbf{C}}(f + ig) = Tf + iTg.$$

The sharp relations between their norms  $\|T\|_{p,q}$  and  $\|T_{\mathbf{C}}\|_{p,q}$  are investigated.

The result can be used in the estimation of the constant in the real Riesz-Thorin interpolation theorem for the first quadrant (that is, for Banach and quasi-Banach Lebesgue spaces).

There are also more general estimates, the so-called *vector-valued inequalities*, that is, the relation between the norm of operator  $T : L^p(\mu) \rightarrow L^q(\nu)$  and its natural vector-valued extension  $T_X : L^p(\mu, X) \rightarrow L^q(\nu, X)$ . Such natural generalizations are connected with the  $n$ -dimensional Grothendieck constants  $K_G(n)$  and the usual real Grothendieck constant  $K_G$ .

## REFERENCES

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