ASYMPTOTIC ESTIMATES FOR APPROXIMATION NUMBERS OF THE HARDY OPERATOR IN q-BANACH SPACE

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Abstract. Let $\mathcal{L}(E,F)$ denote the set of all linear bounded operators $T : E \to F$ from a Banach space E into a q-Banach space F $(0 < q \le 1)$. For any positive integer n, the *n-th approximation number* of $T \in \mathcal{L}(E,F)$ is defined by

$$a_n(T) = \inf\{\|T-L\|_{E\to F} :\ L \in \mathcal{L}(E,F),\ \text{rank } L \le n-1\},$$

where rank L is the dimension of the range of L.

In recent years, a significant amount of attention was paid to the study of the approximation numbers of the Hardy operator $T : L_p(0,\infty) \to L_(0,\infty)$, $1 < p,q < \infty$,

$$Tf(x) = v(x)\int_0^x f(y)\,dy.$$

The exact two-sided asymptotic estimates for approximation and entropy numbers of the Hardy operator in Lebesgue spaces on a semiaxis for $1 < p,q < \infty$ were obtained in monograph [1]. Further, these results were generalized in [2] for one-weight Riemann-Liouville operator of an arbitrary integer order. This work supplement investigations of [1], [3] in the new case $0 < q < 1 < p < \infty$. We set

$$|v|_r = \left(\sum_{k\in\mathbb{Z}} 2^{kr/p'} \left(\int_{2^k}^{2^{k+1}} |v(x)|^q dx\right)^{r/q}\right)^{1/r},$$

and $\frac{1}{r} = \frac{1}{p'} + \frac{1}{q}$. The main result is the following.

Theorem. *Let $0 < q < 1 < p < \infty$, and let $T : L_p(0,\infty) \to L_q(0,\infty)$ be a compact operator. Then, for some constants c_1, c_2 that are either absolute or dependent only on p and q, the following estimates hold:* $\sup\limits_n n a_n(T) \le c_1 |v|_r$, *and if $|v|_r < \infty$, then* $\limsup_{n\to\infty} n\, a_n(T) \le c_2 \left(\int_0^\infty |v(x)|^r dx\right)^{1/r}$.

References

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