

HEPTAGONAL TRIANGLE AS EXTREME TRIANGLE OF DIXMIER-KAHANE-NICOLAS INEQUALITY

ZDENKA KOLAR-BEGOVIĆ¹ AND RUŽICA KOLAR-ŠUPER²

¹DEPARTMENT OF MATHEMATICS, UNIVERSITY OF OSIJEK, CROATIA

²FACULTY OF TEACHER EDUCATION, UNIVERSITY OF OSIJEK, CROATIA

ABSTRACT. Let T be a triangle in a Euclidean plane. Let $g(T)$ be the orthic triangle of the triangle T , and let $g^2(T)$ be the orthic triangle of the triangle $g(T)$; generally let $g^{n+1}(T)$ be the orthic triangle of the triangle $g^n(T)$. In [1] Dixmier, Kahane and Nicolas have proved, by means of trigonometric series, that for $n \rightarrow \infty$ the triangle $g^n(T)$ tends to the point L , a new characteristic point of the triangle T . If (O, R) is the circle circumscribed to the triangle T , then it has been also shown that $|OL| \leq \frac{4}{3}R$ for all triangles T and that $|OL| = \frac{4}{3}R$ if and only if the angles of T are $\frac{4}{7}\pi$, $\frac{2}{7}\pi$, $\frac{1}{7}\pi$. This special triangle is called heptagonal triangle. It is very interesting and rare occurrence that heptagonal triangle is the extreme triangle because the extreme triangle in most of different extreme problems about triangles is equilateral triangle.

It will be proved geometrically that equality in Dixmier–Kahane–Nicolas inequality $|OL| \leq \frac{4}{3}R$ is valid in the case of heptagonal triangle. The relationship between the initial heptagonal triangle T and the obtained point L will also be investigated.

REFERENCES

- [1] J. DIXMIER, J.-P. KAHANE, J.-L. NICOLAS, *Un exemple de non-dérivabilité en géométrie du triangle*, *Ens. Math.* **53**, (2), (2007), 369–428.