

HERMITE-HADAMARD'S INEQUALITY AND THE p - HH -NORM

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ABSTRACT. The Cartesian product of two copies of a normed space is naturally equipped with the well-known p -norm. Another notion of norm is introduced, and will be called the p - HH -norm. This norm is an extension of the generalised logarithmic mean and is connected to the p -norm by the Hermite-Hadamard's inequality. The Cartesian product space (with respect to both norms) is complete, when the (original) normed space is. A proof for the completeness of the p - HH -norm via Ostrowski's inequality is provided. This space is embedded as a subspace of the well-known Lebesgue-Bochner function space (as a closed subspace, when the norm is a Banach norm). Consequently, its geometrical properties are inherited from those of Lebesgue-Bochner space. An explicit expression of the superior (inferior) semi-inner product associated to both norms is considered. Several norm inequalities of Ostrowski type, which involve the p - HH -norm, are also derived using the convexity and the absolute continuity of the norm. Some of these inequalities are proven to be sharp.