ON A MONOTONE ITERATIVE METHOD

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Abstract. We deal with the following boundary value problem:

$$\begin{cases} x'(t) = f(t, x(t), x(\alpha(t))) \equiv Fx(t), \quad t \in J = [0, T],\ T > 0, \\ 0 = g(x(0), x(T)), \end{cases}$$

where $f \in C(J \times \mathbb{R} \times \mathbb{R}, \mathbb{R}),\ \alpha \in C(J, J),\ g \in C(\mathbb{R} \times \mathbb{R}, \mathbb{R})$.

Using the notion of lower and upper solutions we establish general sufficient conditions when problem (1) has extremal solutions or a unique solution in a subset generated by lower and upper solutions. Problem (1) is discussed under the assumption that $t \leq \alpha(t) \leq T$ or $\alpha(t) \leq t$ on J. It means that argument α in problem (1) can be of advanced or delayed types.

To formulate existence results for problem (1) we need to discuss following differential inequalities:

$$\begin{cases} p'(t) \geq K(t)p(t) + L(t)p(\alpha(t)), \ \ t \leq \alpha(t) \leq T, \ \ t \in J, \\ p(T) \leq 0, \end{cases}$$

and

$$\begin{cases} q'(t) \leq -K(t)q(t) - L(t)q(\alpha(t)), \ \ \alpha(t) \leq t, \ \ t \in J, \\ q(0) \leq 0, \end{cases}$$

Functions $K \in C(J, \mathbb{R}),\ L \in C(J, \mathbb{R}_+)$ are corresponding functional coefficients connected with the one–sided Lipschitz condition on f with respect to the last two variables.

References

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