ABOUT ONE INTEGRAL INEQUALITY WITH DELAYS AND ITS APPLICATION

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Abstract. The following result on integral inequalities of Volterra type with delays on the half-interval $J = [t_0, \infty)$ is proven.

Lemma. If 1) $\psi(t) \in C^1(J, \mathbb{R} \setminus \{0\})$, $y(t) \in C^1(J, \mathbb{R})$, $q(t, c) \in C(J \times \mathbb{R}, \mathbb{R}_+)$, $g_k(t) \in C(J, \mathbb{R}_+)$, $\alpha_k(t) \in C(J, \mathbb{R})$, $h_k(t, \tau) \in C(J \times [t_0, t], \mathbb{R}_+)$, $\beta_k(t) \in C(J, \mathbb{R})$ $(k = 1..m)$;
2) (delays) $\alpha_k(t) \le t$, $\beta_k(t) \le t$ $(k = 1..m)$;
3) $\left|\int\limits_{t_0}^{t} \psi(s) y'(s) ds\right| \le q(t, c) + \sum\limits_{k=1}^{m} \int\limits_{t_0}^{t} [g_k(s) \left|y(\alpha_k(s))\right| + \int\limits_{t_0}^{s} h_k(s, \tau) \left|y(\beta_k(\tau))\right| d\tau] ds$, $t \in J$,
then

$$|y(t)| \le |\psi(t)|^{-1} \left\{ q(t, c) + e^{P(t)} \left[c_0 + \int\limits_{t_0}^{t} e^{-P(s)} F(s, c_0, c) ds \right] \right\}, \quad t \in J,$$

where

$$c_0 = |\psi(t_0) y(t_0)|,$$

$$P(t) \equiv \pi(t) + Q(t; 1, \alpha, \beta), \quad \pi(t) \equiv \int_{t_0}^{t} \left|\psi'(s)(\psi(s))^{-1}\right| ds,$$

$$Q(t; q, \alpha, \beta) \equiv \sum_{k=1}^{m} \int\limits_{t_0}^{t} \left[g_k(s) \left|\psi(\alpha_k(s))\right|^{-1} q(\alpha_k(s), c) + \int\limits_{t_0}^{s} h_k(s, \tau) \left|\psi(\beta_k(\tau))\right|^{-1} q(\beta_k(\tau), c) d\tau \right] ds,$$

$$F(t, c_0, c) \equiv \pi'(t) q(t, c) + Q'(t; q, \alpha, \beta).$$

On the base of this result:

1) sufficient conditions of boundedness of solutions with their first derivatives of Volterra weakly nonlinear second order integro-differential equations with delays are obtained;

2) the problem of influence of delays on the boundedness of solutions of weakly nonlinear second order differential equation with respect to corresponding equations without delays is discussed.