

## SOME INEQUALITIES AND GEOMETRY IN BANACH SPACES

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ABSTRACT. Let  $X$  be a Banach space,  $U(X) = \{x \in X, \|x\| \leq 1\}$ , and  $S(X) = \{x \in X, \|x\| = 1\}$  be the unit ball and unit sphere of  $X$  respectively. Let  $\delta(\epsilon)$  and  $C(\epsilon)$ ,  $\rho_X(\epsilon)$ ,  $E(X) = \sup\{\|x + y\|^2 + \|x - y\|^2, x, y \in S(X)\}$ ,  $J(X)$ ,  $O(X)$  and  $Q(X)$ , and  $w(X)$  be the moduli of convexity, the modulus of smoothness, the Pythagorean parameter, the modulus of squareness, moduli of arc lengths, and modulus of W-compactness of  $X$  respectively.

In this talk we show that if the values of these parameters are bounded by some certain numbers, then  $X$  has uniform normal structure therefore has fixed point for a nonexpansive mapping. The values of these parameters in some classical Banach spaces are estimated. Some known results are improved.