ACCURATE APPROXIMATIONS OF THE RIEMANN-STIELTJES INTEGRAL VIA THEORY OF INEQUALITIES

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Abstract. In order to provide *a priori* sharp bounds for the *approximation error* of the *Riemann-Stieltjes integral* $\int_a^b f(t)du(t)$ with various simpler quantities, we consider the functionals:

$$D(f,u;a,b) := \int_a^b f(t)du(t) - \frac{1}{b-a}[u(b)-u(a)] \cdot \int_a^b f(t)dt, \ ([12],\ [13])$$

$$\Theta(f,u;a,b,x) := \int_a^b f(t)du(t) - f(x)[u(b)-u(a)], \ ([4],\ [5])$$

$$T(f,u;a,b,x) := \int_a^b f(t)du(t) - [u(b)-u(x)]f(b) - [u(x)-u(a)]f(a), \ ([11])$$

$$\Sigma(f,u,\varphi,\Phi;a,b) := \int_a^b f(t)du(t) - \frac{\varphi+\Phi}{2} \cdot \int_a^b f(t)dt, \ ([9])$$

where $u : [a,b] \to \mathbb{R}$ is a (φ,Φ)-Lipschitzian function on $[a,b]$, and some other related ones and establish sharp upper bounds for the absolute value of these functionals in terms of the total variation or the Lebesgue norms or the functions involved.

The presentation is both a survey of the results in the published papers below and of some new ones that have been recently obtained by the author.

References

[1] N. S. Barnett, W. S. Cheung, S. S. Dragomir and A. Sofo, *Ostrowski and trapezoid type inequalities for the Stieltjes integral with Lipschitzian integrands or integrators*, Preprint RGMIA Res. Rep. Coll. **9**, 4, (2006), Article 9. [ONLINE: http://rgmia.vu.edu.au/v9n4.html].

[2] P. Cerone, W. S. Cheung and S. S. Dragomir, *On Ostrowski type inequalities for Stieltjes integrals with absolutely continuous integrands and integrators of bounded variation*, Comput. Math. Appl., **54**, 2, (2007), 183–191. Preprint RGMIA Res. Rep. Coll. **9**, 2, (2006), Article 14. [ONLINE: http://rgmia.vu.edu.au/v9n2.html].

[3] W. S. Cheung and S. S. Dragomir, *Two Ostrowski type inequalities for the Stieltjes integral of monotonic functions*, Bull. Austral. Math. Soc., **75**, 2, (2007), 299–311, Preprint *RGMIA Res. Rep. Coll.* **9**, 3, (2006), Article 8. [ONLINE http://rgmia.vu.edu.au/v9n3.html].

[4] S. S. Dragomir, *On the Ostrowski's inequality for Riemann-Stieltjes integral*, Korean J. Appl. Math., **7**, (2000), 477–485.

[5] S. S. Dragomir, *On the Ostrowski's inequality for Riemann-Stieltjes integral* $\int_a^b f(t)\,du(t)$ *where* f *is of Hölder type and* u *is of bounded variation and applications*, J. KSIAM, **5**, 1, (2001), 35–45.

[6] S. S. Dragomir, *Inequalities of Grüss type for the Stieltjes integral*, Kragujevac J. Math., **26**, (2004), 89–122.
[7] S. S. Dragomir, *A generalisation of Cerone's identity and applications*, Oxford Tamsui J. Math., (in press), Preprint RGMIA Res. Rep. Coll. **8**, 2, (2005), Article 19.[ONLINE: http://rgmia.vu.edu.au/v8n2.html].
[8] S. S. Dragomir, *Inequalities for Stieltjes integrals with convex integrators and applications*, Appl. Math. Lett., **20**, (2007), 123–130.
[9] S. S. Dragomir, *Approximating the Stieltjes integral for* (φ, Φ)*-Lipschitzian integrators*, Bull. Austral. Math Soc., **77**, 1, (2008), 73–90.
[10] S. S. Dragomir, *Approximating the Stieltjes integral of bounded functions and applications for three point quadrature rules* Bull. Korean Math. Soc., **44**, (2007), no. 3, 523–536.
[11] S. S. Dragomir, C. Buşe, M. V. Boldea and L. Braescu, *A generalisation of the trapezoidal rule for the Riemann-Stieltjes integral and applications*, Nonlinear Anal. Forum, (Korea) **6**, 2, (2001), 337–351.
[12] S. S. Dragomir and I. Fedotov, *An inequality of Grüss type for the Riemann-Stieltjes integral and applications for special means*, Tamkang J. Math., **29**, 4, (1998), 287–292.
[13] S. S. Dragomir and I. Fedotov, *A Grüss type inequality for mappings of bounded variation and applications for numerical analysis*, Nonlinear Funct. Anal. Appl., **6**, 3, (2001), 425–433.
[14] S. S. Dragomir and A. Sofo, *Approximating the Stieltjes integral via the Darst-Pollard inequality* Filomat **21**, 2, (2007), 63–75.
[15] Z. Liu, *Refinement of an inequality of Grüss type for Riemann-Stieltjes integral*, Soochow J. Math., **30**, 4, (2004), 483–489.