LOGARITHMIC CONVEXITY AND HOW TO PROVE IT

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Abstract. A sequence (a_n) of positive real numbers is logarithmically convex or log-convex if the inequality $a_n^2 \leq a_{n-1}a_{n+1}$ is valid for all $n \geq 1$; it is logarithmically concave or log-concave if we have $a_n^2 \geq a_{n-1}a_{n+1}$ for $n \geq 1$. The last couple of years saw development of new methods for establishing log-convexity or log-concavity of combinatorial sequences defined by linear recurrences. Most of those methods relied on the monotonicity of the quotient sequence $q_n = \frac{a_n}{a_{n-1}}$ of a given sequence (a_n). The monotonicity of quotient sequences is established inductively; however, the nonlinearity of the defining recurrences for (q_n) can be a source of significant technical complications. Here we present a new method that reduces the problem of log-convexity to the problem of establishing the convexity of an associated sequence defined by a linear recurrence.