FIXED POINTS AND GENERALIZED STABILITY FOR FUNCTIONAL EQUATIONS IN ABSTRACT SPACES II

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Abstract. D. H. Hyers in 1941 gave an affirmative answer to a question of Ulam, concerning the stability of group homomorphisms in Banach spaces: *Let E_1 and E_2 be Banach spaces and $f : E_1 \to E_2$ be such a mapping that*

$$\|f(x+y) - f(x) - f(y)\| \leq \delta, \tag{1}$$

*for all $x, y \in E_1$ and a $\delta > 0$, that is f is $\delta-$***additive**. *Then there exists a unique* ***additive mapping*** $T : E_1 \to E_2$, given by

$$T(x) = \lim_{n\to\infty} \frac{f(2^n x)}{2^n}, \ \forall x \in E_1, \tag{2}$$

which satisfies $\|f(x) - T(x)\| \leq \delta, \forall x \in E_1$.

Subsequently, T. Aoki, D. Bourgin and Th. M. Rassias studied the stability problem with unbounded Cauchy differences. Generally, the constant δ in (1) is replaced by a control function, $||\mathcal{D}_f(x,y)|| \leq \delta(x,y)$, where, for example, $\mathcal{D}_f(x,y) = f(x+y) - f(x) - f(y)$ for Cauchy equation. The **stability estimations** are of the form $||f(x) - S(x)|| \leq \varepsilon(x)$, where S **verifies the functional equation** $\mathcal{D}_S(x,y) = 0$, and for $\varepsilon(x)$ explicit formulae are given, which depend on the control δ as well as on the equation.

We use a fixed point method, initiated in [4] and developed in [1] and [3], to give an Ulam-Hyers stability result for functional equations in single variable on random normed spaces. This result is then used to obtain the stability for Cauchy, quadratic and monomial functional equations.

References

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