UNIFORM EXPONENTIAL STABILITY OF AN OPERATOR SEMIGROUP ON HILBERT SPACE AND AN INFINITE TIME CAUCHY PROBLEM DRIVEN BY ITS GENERATOR

CONSTANTIN BUŞE

Department of Mathematics, Faculty of Mathematics and Computer Science, West University of Timişoara, România

Abstract. Let $H = L^2[0,\pi]$ and $A : D(A) \subset H \to H$ be given by $Ax = \frac{d^2x}{d\xi^2}$. Here the domain $D(A)$ consists of all absolutely continuous functions $x(\cdot)$ defined on $[0,\pi]$, which satisfy the following three conditions:

i) $x(0) = x(\pi) = 0$.

ii) The first derivative $\frac{dx}{d\xi}$ is absolutely continuous on $[0,\pi]$.

iii) The second derivative $\frac{d^2x}{d\xi^2}$ belongs to H.

We prove that if Φ is a certain nondecreasing and convex function on $\mathbf{R}_+ := [0,\infty)$ and L^Φ is the associated Orlicz space then for each $u^* \in (L^\Phi)^*$ and each $b \in H$, the infinite Cauchy Problem

$$\begin{aligned} &\frac{\partial y(t,\xi)}{\partial t} = \frac{\partial^2 y(t,\xi)}{\partial \xi^2} + u^*(-t)b(\xi) \quad \text{for } t \in (\infty, 0],\ \xi \in (0,\pi) \\ &\lim_{t\to-\infty} \int_0^\pi |y(t,\xi)|^2 d\xi = 0 \end{aligned}$$

has a unique solution.