

# THE HERMITE–HADAMARD INEQUALITY IN BECKENBACH'S SETTING

MIHÁLY BESSENYEI

UNIVERSITY OF DEBRECEN, HUNGARY

ABSTRACT. The classical Hermite–Hadamard inequality, as is well known from numerical analysis, provides a lower and an upper estimation for the integral average of convex functions defined on compact intervals, involving the midpoint and the endpoints of the domain. The aim of the talk is to extend this inequality to the case when the convexity notion is induced by a Beckenbach family.

A Beckenbach family is a set of continuous functions and is determined by the property that prescribing certain points on the plain (with pairwise distinct first coordinates) there exists precisely one member of the family that interpolates the points. Applying Beckenbach families, the classical convexity notion can be considerably generalized. The obtained convexity notion involves the notion of higher-order convexity due to Popoviciu; or, in more general, the convexity notion induced by Chebyshev systems.

In our approach, the geometry of Beckenbach families plays a key role, guaranteeing the existence of so-called principal supports for a given generalized convex function. The extension of the Hermite–Hadamard inequality is a direct consequence of the support properties. The Markov–Krein representation problem of Beckenbach families is also investigated.