

SUPERSTABILITY OF SOME CONDITIONAL CAUCHY FUNCTIONAL EQUATIONS AND RELATED QUESTIONS

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ABSTRACT. We will be concentrated on certain conditional Cauchy equations with the condition dependent on the unknown function. Consider, for instance, Dhombres' functional equation

$$(f(x) + f(y)) \cdot (f(x + y) - f(x) - f(y)) = 0. \quad (1)$$

This equation is superstable in the sense of Baker, i.e. the only unbounded solution of the inequality

$$|(f(x) + f(y)) \cdot (f(x + y) - f(x) - f(y))| \leq \varepsilon, \quad x, y \in G,$$

with given $\varepsilon > 0$, in the class of complex functions defined on an abelian group $(G, +)$, is an exact solution of Dhombres' equation (1).

Motivated by the case of the exponential functional equation we deal with approximate solutions of Dhombres' equation, rewritten in the form

$$f(x + y)(f(x) + f(y)) = (f(x) + f(y))^2, \quad (2)$$

defined as functions with the quotient of the left hand side and the right hand side of equation (2) close to 1.