BEST CONSTANT IN TRIANGLE INEQUALITY IN LORENTZ SPACES

SORINA BARZA

Karlstad University, Sweden

Abstract. The study of the normability of the Lorentz spaces $L^{p,s}(R,\mu)$ goes back to G.G. Lorentz. The condition defining these spaces is given in terms of the non-increasing rearrangement of f:

$$\|f\|_{p,s} = \left(\int_0^\infty \left(t^1/pf^*(t)\right)^s \frac{dt}{t} \right)^{1/s},$$

with the usual modification if $s = \infty$. G.G. Lorentz proved that $\| \cdot \|_{p,s}$ is a norm if and only if $1 \leq s \leq p < \infty$ and the space $L^{p,s}(R,\mu)$ is normable (i.e. there exists a norm equivalent to $\| \cdot \|_{p,s}$) for the range $1 < p < s \leq \infty$. The fact that $\| \cdot \|_{p,s}$ is normable is equivalent with the fact that it satisfies the triangle inequality uniformly on the number of terms i.e. there exists a constant $c_{ps} > 0$ such that for every finite collection $\{f_k\}_{k=1 \ldots N}$:

$$\| \sum_{k=1}^N f_k \|_{p,s} \leq c_{ps} \sum_{k=1}^N \|f_k\|_{p,s}.$$

Based on the results in [1] we show that the best constant in the above inequality is $c_{ps} = \left(\frac{p}{s}\right)^{1/s} \left(\frac{p'}{s'}\right)^{1/s'}$.

References

[1] S. Barza, V. Kolyada and J. Soria, *Sharp constants related to the triangle inequality in Lorentz spaces*, Transactions of AMS, to appear.