HISTORY AND NEW REVEALS ON ALZER'S INEQUALITY

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Abstract. In 1993. Horst Alzer proved the following inequality

$$\frac{n}{n+1} \leq \left[\frac{(n+1)\sum\limits_{i=1}^{n} i^r}{n\sum\limits_{i=1}^{n+1} i^r} \right]^{\frac{1}{r}},$$

for $n \in \mathbb{N}$, $r \in \mathbb{R}^+$. Since then this inequality is known as Alzer's inequality. However, in 1975. Jan Van de Lune stated *Problem 399.*, which was solved by several mathematicians, and which easily implicates Alzer's inequality. We will show that, what is called "Alzer's inequality", is actually the result of J. Van de Lune's work and according to that, the above inequality will be called Van de Lune-Alzer's inequality. We will give a review of different methods in proving and generalizing Van de Lune - Alzer's inequality as well as our corrections, refinements and extensions of some results and proofs. New results, inspired by the generalization of Van de Lune - Alzer's inequality for increasing convex sequences presented by N. Elezović and J. Pečarić, will be shown.