MAPPINGS CONNECTED WITH HERMITE-HADAMARD INEQUALITIES FOR SUPERQUADRATIC FUNCTIONS

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Abstract. Let $\varphi : [0,\infty) \to \mathbb{R}$ be a continuous superquadratic function, $g : [a,b] \to \mathbb{R}$ be a nonnegative continuous function, and let $p : [a,b] \to \mathbb{R}$ be a positive integrable function. We consider the following two mappings $H, F : [0,1] \to \mathbb{R}$, defined as

$$H(t) = \frac{1}{P}\int_a^b p(x)\,\varphi(tg(x) + (1-t)\,\overline{g})\,dx$$

and

$$F(t) = \frac{1}{P^2}\int_a^b \int_a^b p(x)\,p(y)\,\varphi(tg(x) + (1-t)\,g(y))\,dxdy$$

where

$$P = \int_a^b p(x)\,dx, \qquad \overline{g} = \frac{1}{P}\int_a^b p(x)\,g(x)\,dx,$$

which were introduced and considered by Y. J. Cho, M. Matić and J. Pečarić in [2] for the convex function φ. Convexity and some other properties have been established for these two mappings.

In distinction from the convex case, superquadracity is not hereditary. If φ is superquadratic function, mappings H and F are not superquadratic in general, but we established some other properties and inequalities which refine some results from [2].

In the special case for $p(x) \equiv 1$ and $g(x) = x$ $(x \in [a,b] \subseteq [0,\infty))$ we get refinements of some results of S. S. Dragomir [3] and some results of M. Akkouchi [1].

References

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[3] S. S. Dragomir, *Two mappings in connection to Hadamard's inequalities*, J. Math. Anal. Appl. 167 (1992), no. 1, 49–56.