

ON TWO VARIABLE FUNCTIONAL INEQUALITY AND RELATED FUNCTIONAL EQUATION

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ABSTRACT. We present the result stating that the lower semicontinuous solutions of a large class of functional inequalities can be obtained from particular solutions of the related functional equations. Our main theorem reads as follows.

Theorem. Let $\lambda : I^2 \rightarrow (0, 1)$ be a function and $n, m : I^2 \rightarrow I$ be continuous strict means. If there exists a non-constant and continuous solution $\phi : I \rightarrow \mathbb{R}$ of equation

$$T_{(n(x,y),m(x,y))}^\lambda \phi = T_{(x,y)}^\lambda \phi, \quad x, y \in I,$$

then ϕ is one-to-one, and a lower semicontinuous function $f : I \rightarrow \mathbb{R}$ satisfies inequality

$$T_{(n(x,y),m(x,y))}^\lambda f \leq T_{(x,y)}^\lambda f, \quad x, y \in I,$$

if and only if $f \circ \phi^{-1}$ is convex on $\phi(I)$.

This result improves results presented in [1] and [2].

REFERENCES

- [1] J. MATKOWSKI AND M. WRÓBEL, *A generalized a -Wright convexity and related function equation*, Ann. Math. Silesianae 10 (1996), 7-12.
- [2] ZS. PÁLES, *On two variable functional inequality*, C. R. Math. Rep. Acad. Sci. Canada, Vol. 10 (1988), 25-28.