INEQUALITIES INVOLVING THE KHATRI-RAO PRODUCT OF MAPS ON POSITIVE DEFINITE MATRICES

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Abstract. We shall show several complementary inequalities to Jensen's type inequality involving the Khatri-Rao product of maps on positive definite matrices, which is based on the Mond-Pečarić method. In particular, we determine real constants α_1, α_2, β_1 and β_2 such that

$$\alpha_2\left(\mathop{\ast}_{i=1}^{n}\Phi_i(A_i^s)\right)^{1/s}\le\left(\mathop{\ast}_{i=1}^{n}\Phi_i(A_i^r)\right)^{1/r}\le\alpha_1\left(\mathop{\ast}_{i=1}^{n}\Phi_i(A_i^s)\right)^{1/s}$$

and

$$\beta_2 I\le\left(\mathop{\ast}_{i=1}^{n}\Phi_i(A_i^s)\right)^{1/s}-\left(\mathop{\ast}_{i=1}^{n}\Phi_i(A_i^r)\right)^{1/r}\le\beta_1 I$$

hold if $r\le s$, $r,s\neq 0$, where A_i $(i=1,\dots,n)$ are compatibly partitioned positive definite matrices, Φ_i are normalized positive linear maps, and $\mathop{\ast}_{i=1}^{n}A_i$ denote the Khatri-Rao product $A_1*A_2*\cdots*A_n$.